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Reg. No. : ....

Name : ....

Fourth Semester B.Tech. Degree Examination, April/May 2012
(2008 Scheme)
08.401 : ENGINEERING MATHEMATICS – III (CMPUNERFHB)

Time : 3 Hours Max. Marks : 100

Instructions : *Answer **all** questions from Part **A, each** question carries **four** marks and **one** full question from **each** Module of Part **B each** full question carries **twenty** marks.*

PART – A

1. Show that $f(z) = \sqrt{|xy|}$ satisfies Cauchy Riemann equations at the origin but not differentiable at origin.

2. Show that f(z) = log z is differentiable except at z = 0 and find its derivative.

3. Show that under the transformation $w = z^2$, the first quadrant of the z-plane is mapped onto the upper half of w-plane.

4. Evaluate $\int_c \text{Re}(z)\, dz$ where c is the shortest path from 1 + i to 3 + i.

5. State Cauchy's integral theorem. Is the converse true ? Give an example.

6. Evaluate $\int_c \frac{1}{z^2+9}\, dz$ where c is $|z - 3i| = 4$.

7. Find the poles and residues of $f(z) = \frac{1-e^{2z}}{z^4}$.

8. Given that $\int_0^{\frac{1}{4}} e^{x^2}\, dx = 0.2553074606$. Find the truncation error if

$e^{x^2} \cong 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!}$.

P.T.O.

9. Find by Newton-Raphson method the real root of $\log_e x - \cos x = 0$. Correct to 3 places of decimals.

10. Use Lagrange's interpolation formula to find y (– 2) given

x :	– 1	0	2	3
y :	– 8	3	1	2

PART – B

MODULE – I

11. a) If u and v are harmonic, prove that $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ is analytic.

b) Find the analytic function f(z) = u + iv if $u + v = \frac{x}{x^2 + y^2}$ given f(1) = 1.

c) Find the image of the strip $\frac{1}{4} \le y \le \frac{1}{2}$ under the transformation $w = \frac{1}{z}$.

12. a) Prove that an analytic function f(z) is independent of $\bar{z}$.

b) Show that e^x (x cos y – y sin y) is harmonic. Find the analytic function for which e^x (x cos y – y sin y) is the imaginary part.

c) Determine the bilinear transformation that sends the points z = 0, – i, 2 i into the points w = 5i, ∞, $-\frac{i}{3}$ respectively. Find the invariant points and the image of | z | < 1.

MODULE – II

13.a) Evaluate $\int_c (z^2 + 3z)\,dz$ along the circle | z | = 2 from (2, 0) to (– 2, 0) in the counter clockwise direction. Without integration is it possible to predict the value of the integral for the same circle | z | = 2 from (– 2, 0) to (2, 0). Give reasons.

b) Find the Laurent series expansion of $f(z) = \dfrac{7z^2+9z-18}{z(z+3)(z-3)}$ valid for

i) $0 < |z| < 3$ ii) $|z| > 3$.

c) Evaluate $\int_0^{\pi} \dfrac{\cos 2\theta}{5+4\cos\theta}\, d\theta$.

14. a) Evaluate $\int_C \dfrac{e^z}{z(1-z)^3}\, dz$ where C is $|z-1| = \dfrac{1}{2}$.

b) Find the residue of $f(z) = \dfrac{\sin z}{z^4}$ and hence evaluate $\int_{|z|=1} \dfrac{\sin z}{z^4}\, dz$.

c) Evaluate $\int_{-\infty}^{\infty} \dfrac{x^2}{(x^2+a^2)^3}\, dx$.

MODULE – III

15. a) Solve $x^3 - 9x + 1 = 0$ for the root lying between 2 and 4 by regula falsi method correct to two decimals.

b) Use Newton's interpolation formula to find y(17) given the following table :

x :	8	10	12	14	16	18
y :	10	19	32.5	54	89.5	154

c) Find the approximate value of $\int_0^1 \dfrac{1}{1+x^2}\, dx$ by

i) Trapezoidal rule and

ii) Simpson's $\dfrac{1}{3}$ rule. Take $h = 0.1$.

16. a) Find a root of $x^3 - x - 11 = 0$ that lies between 2 and 3 correct to two decimal places by bisection method.

b) Apply Gauss-Seidal method to solve the system of equations

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$

$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$

$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$

$$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$

c) Use Euler method to solve $\frac{dy}{dx} = \frac{y-x}{y+x}$ with initial condition

$y = 1$ at $x = 0$; for $x = 0.1$. Take $h = 0.02$.
